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BY

BERTRAM G. COOPER, A.F.AË.S.

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Notices of the Aëronautical Society of Great Britain.

Members' Subscriptions.

Members who have not yet paid their subscriptions for the current year are reminded that these became due on the 1st January.

Times of Opening of Society's Offices.

The Society's Offices are now open daily from 10.0 a.m. to 5.0 p.m., except on Saturdays, when they are open from 10.0 a.m. to 12.30 p.m.

Reviews and Obituary.

Temporarily held over.

Aero Educational Premiums Fund.

The following account shows the state of the fund to the 31st December, 1915:—

Cr. 1915.

		£	s.	d.
Jan. 1.	Balance at Bank	21	16	11
Jan. 1.	£600 3½% War Loan (G. Brewer and others) at cost	566	11	7
	Total of fund	588	8	6

Dr. 1915.

	Investment as above	566	11	7
	Balance at Bank, 31st December, 1915...	£21	16	11

Wilbur Wright Memorial Fund.

The following account shows the state of the fund to the 31st December, 1915:—

Cr. 1915.

			£	s.	d.
Jan.	1.	Balance brought forward	121	5	1
Jan.	1.	Canada 4% Stock, 1940-60 (G. Brewer and others) at cost	531	10	9
Apl.	3.	Dividend on £550 Canada 4% Stock	9	17	1
Oct.	6.	Dividend on £550 Canada 4% Stock	9	12	6
Dec.	31.	Dividends from Aero Educational Premiums Fund—			
		Mar. 1, 1915 ... ... £1 4 9			
		Sept. 1, 1915 ... ... 8 2 4			
			9	7	1
			£681	12	6

Dr. 1914.

	£	s.	d.
Premium to Prof. G. H. Bryan for Third Wilbur Wright Memorial Lecture	21	0	0
Total of fund	660	12	6
Investment as above	531	10	9
Balance at Bank, 31st December, 1915	£129	1	9

THE CHARACTERISTICS OF CERTAIN RESISTING SURFACES AND SCREWS.

BY G. H. BRYAN, SC.D., F.R.S.

PART I.—CAPTAIN FERBER'S REPRESENTATION BY ORTHOGONAL SYSTEMS OF PLANES.

1. In the stability investigations which the late Captain Ferber published in the *Revue d'Artillerie*, the sustaining and other surfaces of an aeroplane were in certain cases taken to be represented, for dynamical purposes, by a system of three plane resisting laminæ fixed mutually at right angles. Unfortunately, however, such a system cannot in general be made equivalent to a collection of surfaces such as those of an aeroplane, with the result that Captain Ferber's investigation failed to give the correct conditions of lateral stability. At the same time, Ferber's system of three orthogonal planes is so convenient, especially for forming a general idea of the effects of wind gusts on an aeroplane, that it is desirable to investigate conditions and limitations under which such a representation is valid. The desirability of a further investigation of the forces and couples acting on a system of resisting surfaces of a general character was foreshadowed in "*Stability in Aviation*," and a more detailed discussion of the problem has now become necessary in order to prepare the way for further studies in the rigid dynamics of the motions of aeroplanes or of systems resembling them. The present note is necessarily of a somewhat academic character, but it is only necessary to repeat what I have been urging for the last 18 years, that problems bearing on aviation should have been studied in the first instance by mathematicians and the results published in mathematical journals. Instead of this aeroplanes have been practically boycotted by the orthodox mathematical world.

The present paper deals with the properties of a system of rigidly connected non-interfering surface elements moving in a resisting medium and obeying the sine law of resistance. The application of the results to the specification of the forces and couples acting on an actual aeroplane is therefore subject to certain limitations which are clearly understood; in particular the surfaces of an aeroplane must partake of the character of "narrow planes gliding at small angles" in order that the results may become applicable as first approximations. When a given system is replaced by a system of three or more laminæ in three mutually perpendicular planes, the latter are *assumed* to be subject to the sine law of resistance for *all* values of the angle of attack, but as the new system is a purely hypothetical one, this should not occasion any difficulty. Similar assumptions are made in the solution of problems in all other branches of mathematical physics. The sine law is particularly convenient for mathematical purposes, since the pressure on a lamina is proportional to the product of the resultant relative air velocity into its component normal to the lamina.

2. Consider then a surface element S and let λ, μ, ν, be the direction cosines of its normal, l, m, n, the direction cosines of the relative wind, V the velocity of the latter. The pressure on the element is by assumption

$$KV^2S\ (l\lambda + m\mu + n\nu) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and the x component of the resultant pressure on a system of such elements is

$$V^2\left\{l\Sigma K\lambda^2 S + m\Sigma K\lambda\mu S + n\Sigma K\lambda\nu S\right\} \quad . \quad . \quad . \quad . \quad (2)$$

Hence the *drag* or resolved part of the resultant pressure in the direction in which the wind is blowing is given by the formula

$$D/V^2 = Al^2 + Bm^2 + Cn^2 + 2Fmn + 2Gnl + 2Hlm$$
$$\text{where } A = \Sigma K\lambda^2 S,\ F = \Sigma K\lambda\mu S \quad . \quad . \quad . \quad . \quad (3)$$

and similar expressions for the other coefficients.

Now consider the conicoid

$$Ax^2 + By^2 + Cz^2 + 2Fyz + 2Gzx + 2Hxy = E \quad . \quad . \quad . \quad (4)$$

where E is any constant of suitable dimensions. If r be the radius vector of this conicoid in the direction (l, m, n) of the wind, the drag is given by

$$D = EV^2/r^2 \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The conicoid must be an ellipsoid, for otherwise D would be negative for certain directions of the wind, in other words, the system would tend to fly against the wind, which is inconsistent with the principles of conservation and degradation of energy.

Now let the ellipsoid be referred to its principal axes. Then F, G and H vanish and the components of the resultant pressure along the axes are

$$R_x = AlV^2 \quad R_y = BmV^2 \quad R_z = CnV^2 \quad . \quad . \quad . \quad (6)$$

It follows therefore that every resisting system of this kind possesses *three principal axes at right angles such that a wind blowing in the direction of one of these axes produces no component pressure at right angles to that axis.* We might perhaps describe these axes as " axes of no lift " since the lift of an aeroplane represents the component pressure perpendicular to the wind velocity.

The equation of the ellipsoid becomes

$$Ax^2 + By^2 + Cz^2 = E \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and the direction cosines of the perpendicular on the tangent plane at the point (x, y, z) whose direction cosines are (l, m, n) are proportional to Al, Bm, Cn, and therefore to R_x, R_y, R_z, in (6).

It follows that *when the wind blows in the direction of a radius vector of any point of the ellipsoid, the resultant pressure is in the direction of the perpendicular on the tangent plane at that point,* or, what comes to the same thing, in the direction of the normal at that point.

Now let R be the resultant thrust, θ the angle between the radius vector and the perpendicular on the tangent, p the length of the perpendicular on the tangent. Then we have

$$D = R\cos\theta = Rp/r, \text{ and therefore by (5)}$$
$$R = EV^2/pr \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Hence *the resultant thrust is inversely proportional to the product of the radius vector of the ellipsoid into the perpendicular on the tangent plane.*

Finally, if we now put

$$K_1S_1 = A = \Sigma K\lambda^2 S,\ K_2S_2 = B = \Sigma K\mu^2 S,\ K_3S_3 = C = \Sigma K\nu^2 S,$$

we get from (6)

$$R_x = K_1S_1lV^2 \quad R_y = K_2S_2mV^2 \quad R_z = K_3S_3nV^2 \quad . \quad . \quad . \quad (9)$$

so that so far as the magnitude and direction of the resultant thrust is concerned the system is equivalent to three plane laminæ, S_1, S_2, S_3, mutually at right angles to each other; a result so far in accordance with Ferber's assumption.

It now remains to examine whether by suitably choosing the positions of the equivalent plane laminæ the couples about the axes can be made equivalent in the two systems. We assume that the elements are so small that the displace-

ments of their centres of pressure may be neglected, so that if x, y, z, are components of the centre of pressure of the element S of the original system, the couples L, M, N, about the axes are given by equations of the form

$$L = V^2 \Sigma KS \ (y\nu - z\mu) \ (l\lambda + m\mu + n\nu) \quad . \quad . \quad . \quad (10)$$

If (x_1, y_1, z_1) (x_2, y_2, z_2) (x_3, y_3, z_3) are the co-ordinates of the centres of pressure of the equivalent plane laminæ, S_1, S_2, S_3, the corresponding expression for the couple is

$$L = y_3 K_3 S_3 n V^2 - z_2 K_2 S_2 m V^2 \quad . \quad . \quad . \quad (11)$$

If these be equal for all values of l, m, n, we must have

$$\Sigma KS \ (y\lambda\nu - z\lambda\mu) = 0 \text{ and two similar equations} \quad . \quad . \quad (12)$$

$$y_3 K_3 S_3 = \Sigma KS \ (y\nu^2 - z\mu\nu)$$
$$z_2 K_2 S_2 = \Sigma KS \ (z\mu^2 - y\mu\nu) \quad . \quad . \quad . \quad . \quad (13)$$

and two other pairs of similar equations.

Equations (12) require that if $m = 0$ and $n = 0$, L must vanish. In other words, *a wind blowing in the direction of a principal axis must produce no couple about that axis in order that the systems may be equivalent.* This requires that the arrangement of the surfaces of the original system about the principal axes should not partake of the general character of a screw propeller or the sails of a windmill.

The three necessary conditions (12) take the symmetrical form

$$\Sigma KSx\mu\nu = \Sigma KSy\nu\lambda = \Sigma KSz\lambda\mu \quad . \quad . \quad . \quad . \quad (12a)$$

If one of the principal planes is a plane of symmetry of the system each of these expressions vanishes and the necessary conditions are satisfied. It is not necessary that the system should have more than one plane of symmetry, and the results would therefore be applicable to any system resembling an aeroplane.

When these conditions are satisfied the remaining conditions (13) give six equations determining the six co-ordinates (y_1, z_1), (z_2, x_2) and (x_3, y_3), and showing that it is now possible to make the system of orthogonal planes S_1, S_2, S_3, equivalent to the original system by suitably choosing their positions. The co-ordinates x_1, y_2, z_3, do not occur in these equations because if a lamina is displaced perpendicularly to itself the magnitude and position of the resultant thrust on it is unaltered.

We may therefore make x_1, y_2, z_3, each = 0, thus placing the orthogonal laminæ in the co-ordinate planes. We may, moreover, shift the origin so as to make some of the other co-ordinates vanish, thus we may by a suitable choice of origin make x_2, y_3, z_1, each zero. The system then reduces to three laminæ S_1 in the plane of yz with centre on the axis of y at $(0, y_1, 0)$, S_2 in the plane of zx with the centre on the axis of z at $(0, 0, z_2)$, S_3 in the plane of xy with centre on the axis of x at $(x_3, 0, 0)$.

When this is done we must have for the original system

$$\left.\begin{aligned} \Sigma KS \ (\nu y - \mu z) \ \nu = 0 \\ \Sigma KS \ (\lambda z - \nu x) \ \lambda = 0 \\ \Sigma KS \ (\mu x - \lambda y) \ \mu = 0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (14)$$

In this case L is proportional to m, M to n and N to l. Thus, if $l = 0$ then $N = 0$, in other words, *a wind blowing in the plane of yz produces a couple whose axis is in the plane of xy, and generally if the wind blows in one principal plane the axis of the resulting couple is in another principal plane.*

Again, if $l=0$ and $m=0$ then L and N vanish, that is, *a wind in the direction of the axis of z produces a couple about the axis of y*, and *generally if the wind blows in the direction of one principal axis the resulting couple is about another principal axis.*

If the plane of xy is a plane of symmetry we also have $z_2=0$ so that the plane S_2 may now be placed at the origin, or we may make x_1, y_1, z_1, x_2, y_2, z_2 and $z_3=0$, in this case the system will reduce to two plane laminæ S_1 S_2 at right angles about the origin and a third lamina S_3 in the plane of (x_1y) about the point $(x_3, y_3, 0)$.

In this case $N=0$ or *the couple always lies in the plane of xy*, also if $n=0$ then L and M also vanish, so that *if the wind blows in the plane of symmetry the resultant pressure always passes through the origin.*

If the system is equivalent to three planes having their centres at the same point, that point may be taken as the origin and in this case we should have to have $L=0$, $M=0$, $N=0$ always. This would require that *the resultant thrust should always pass through a fixed point.*

4. It will thus be seen that a system having one plane of symmetry may be represented by Captain Ferber's equivalent system of three orthogonal planes for all variations of pressure depending on *motions of translation* of the system relative to the resisting medium, and that the same result holds good for any such system provided that motion in the direction of a principal axis produces no couple about that axis.

When, however, the system has a motion of *rotation* in addition to one of translation the forces and couples on it are no longer equivalent to those on a system of *three* orthogonal planes S_1, S_2, S_3, and failure to realise this fact was one of the causes which led Captain Ferber to obtain incorrect conditions of lateral stability.

Mr. Selig Brodetsky has investigated the conditions under which the system may be replaced, not by *three* orthogonal planes, but by *six* surfaces occurring in pairs, each pair being perpendicular to one of the co-ordinate axes. Even in the simplest cases the algebraic formulæ are very long and cumbersome, involving in one case a group of 36 equations. In order that this mode of representation may be possible Mr. Brodetsky finds that the following conditions must be satisfied:—

1. The components of angular velocity must be small so that their squares and products may be neglected.

2. The direction of the translational motion must nearly coincide with a principal axis so that for purposes of approximation it is possible to take $l=1$ and neglect squares and products of m and n when the axis in question is the axis of x.

These two conditions are eqivalent to the assumption that the components of linear and angular velocity are of the form

$$U+u, v, w, p, q, r,$$

where U is large and the rest are small.

Under certain conditions Mr. Brodetsky finds that the six surfaces can be reduced in number to five, four, or three. It will, however, be seen that *when rotations are taken into account, the representation of the system by any equivalent system of orthogonal planes is only possible in the case of small oscillations or other disturbances about a state of steady motion in a direction nearly coinciding with a principal axis.*

On the other hand the system S_1, S_2, S_3, *does* come in useful in enabling us to determine the *initial accelerations* set up in the system by a sudden gust of wind and it is with a view to this object that the present paper was written.

PART II.—RESISTANCE DERIVATIVES OF SCREWS.

5. Consider a right-handed screw propeller and let its axis of rotation be chosen as axis of x, its centre being the origin. If the propeller be viewed from a point on the axis of y and two of the blades project in the direction of the axis of z in a system of rectangular axes, it will be seen that these blades behave as a left handed screw in relation to the axis of y. It hence follows that if the propeller receive a translational displacement about an axis Oy perpendicular to its axis of rotation, a negative couple is set up about Oy and conversely a rotation about Oy gives rise to a negative thrust along Oy. Using my notation for resistance derivatives it follows that the propeller gives rise to the following coefficients:

$$\begin{array}{cccccc} X_u & Y_v & Z_w & L_p & M_q & N_r \\ X_p & Y_q & Z_r & L_u & M_v & N_w \end{array}$$

and as we always make the convention that these are to be taken positive when they tend to *retard* motions in the positive sense it will readily be seen that those in the first line are in general positive, whereas in the second line X_p and L_u are negative Y_q Z_r M_v N_w being positive.

6. A qualitative estimate of the magnitudes of these derivatives can be obtained by assuming the blades to be "narrow surfaces gliding at small angles," *i.e.*, by supposing that they can be regarded as built up of non-interfering surface elements the resistance on which follows the sine law. The possibility of treating a blade as the sum of its elements is discussed in Lanchester's book, but it would appear both from this and from the writings of Greenhill and others that not only are many theories of air-screw action based on such an assumption, but some of the older ones are based on Newton's "sine squared law" which has been shown to fail for aeroplanes and is probably further from the truth than the sine law in the case of air-screws.

Let U be the forward translational velocity, Ω the angular velocity of the air-screw. Consider an element dS of a blade, let ρ be its distance from the axis, θ its inclination to the plane of yz and ϕ the angle which its radius vector ρ makes with the axis of y, the pitch of the screw of which it forms part thus being $\rho \tan \theta$ and the velocity of slip being $\Omega\rho \tan \theta - U$.

If V be the resultant velocity of the element relative to the air, γ its inclination to the plane of yz, we have

$$V^2 = U^2 + \Omega^2\rho^2, \ \tan \gamma = U/\Omega\rho, \ V \sin \gamma = U, \ V \cos \gamma = \Omega\rho \quad . \quad (15)$$

and α the angle of attack is equal to $\theta - \gamma$.

The pressure RdS on the element is given by

$$RdS = KdSV^2 \sin \alpha = KVdS\,(\Omega\rho \sin \theta - U \cos \theta) \quad . \quad . \quad (16)$$

and the thrust and couple due to the reaction of the air are given by

$$H = \int RdS \cos \theta \qquad G = -\int RdS\rho \sin \theta \quad . \quad . \quad (17)$$

If the screw is of uniform pitch P, so that $\rho \tan \theta$ is constant and equal to P, we have obviously $G = -HP$ and this result is of course quite independent of any assumption as to the nature of the air resistance.

It will be found that

$$\frac{dH}{dU} = \int K dS\,(2U \sin\alpha - \rho\cos\alpha)\cos\theta \quad . \quad . \quad . \quad . \quad (18)$$

$$\frac{dH}{d\Omega} = \int K\rho dS\,(2\Omega\rho \sin\alpha + U\cos\alpha)\,\rho\cos\theta \quad . \quad . \quad . \quad . \quad (19)$$

and dG/dU, $dG/d\Omega$ represent similar expressions with the factor $-\rho\sin\theta$ substituted for $\rho\cos\theta$.

These expressions with their signs changed would represent the resistance derivatives X_u, X_p, L_u, L_p, due to small impressed components of linear and angular velocity u p respectively. They would therefore be the coefficients which would have to be introduced into the equations for small oscillations and stability *if the air-screw were assumed to be revolving with constant angular velocity relative to the machine*. If, however, the engine produces a constant couple on the propeller, then with uniform pitch both H and G will have to remain constant and there will be no resistance derivatives of the forms X_u, X_p, L_u, L_p. The question is therefore one for the engineer and not the mathematician to discuss, and in any case our object is to concentrate our attention on the other resistance derivatives which from considerations of symmetry evidently satisfy the relations

$$Y_v = Z_w, \quad Y_q = Z_r, \quad M_v = N_w, \quad M_q = N_r \quad . \quad . \quad . \quad (20)$$

7. Let a velocity v be imparted to the system in the direction of the axis of y. This produces a radial velocity $v\cos\phi$ and a tangential velocity $-v\sin\phi$ of the element dS in the plane of xy. The increase in the resultant velocity relative to the air is to the first order $-v\sin\phi\cos\gamma$, and of the normal velocity $-v\sin\phi\sin\theta$. Hence the pressure on the element is *decreased* by

$$KdS\,(Vv\sin\phi\sin\theta + Vv\sin\alpha\sin\phi\cos\gamma)$$

that is by

$$KdSVv\sin\phi\,(2\sin\alpha\cos\gamma + \cos\alpha\sin\gamma)$$

or

$$KdSv\sin\phi\,(2\Omega\rho\sin\alpha + U\cos\alpha) \quad . \quad . \quad . \quad (21)$$

Calling this expression for shortness

$$KdSv\sin\phi R_v$$

we see that it gives rise to a force in the negative direction of the axis of y of amount

$$KdSv\sin\phi R_v\sin\theta\sin\phi$$

and a couple about the axis of y of negative moment

$$KdSv\sin\phi R_v\cos\theta\times\rho\sin\phi$$

As the blade revolves round the axis of x the mean value of $\sin^2\phi$ is $\frac{1}{2}$, and in this way we obtain for the resistance derivatives the final forms

$$Y_v = \tfrac{1}{2}\int K dS\,(2\Omega\rho\sin\alpha + V\cos\alpha)\sin\theta \quad . \quad . \quad . \quad . \quad (22)$$

$$M_v = \tfrac{1}{2}\int K dS\,(2\Omega\rho\sin\alpha + V\cos\alpha)\,\rho\cos\theta \quad . \quad . \quad . \quad (23)$$

and it will be observed that $\rho\cot\theta$ is the pitch of the element dS considered as part of a screw revolving about an axis *in the plane* of yz perpendicular to the radius vector ρ, as it should be.

8. Next let the system receive a small rotation q about the axis of y. This produces in the element dS a velocity $q\rho \sin\phi$ in the positive direction of the axis of x, the resulting velocity in that direction being increased from U to $U + q\rho \sin\phi$. Proceeding as before, we find that the pressure on the element dS is now increased by

$$KdSq\rho \sin\phi\,(2U \sin\alpha - \Omega\rho \cos\alpha) \quad . \quad . \quad . \quad (24)$$

and we get for the resistance derivatives

$$Y_q = \tfrac{1}{2}\int KdS\,(\Omega\rho \cos\alpha - 2U \sin\alpha)\,\rho \sin\theta \quad . \quad . \quad . \quad (25)$$

$$M_q = \tfrac{1}{2}\int KdS\,(\Omega\rho \cos\alpha - 2U \sin\alpha)\,\rho^2 \cos\theta \quad . \quad . \quad . \quad (26)$$

Finally the gyrostatic reaction of the air-screw is equivalent in its effect to resistance derivatives of the form

$$M_r = -N_q = \text{angular momentum} \div g \quad . \quad . \quad . \quad (27)$$

In certain cases it is theoretically possible to effect some kind of balance between these components. For suppose the system is turning about an axis parallel to the axis of y at $x = a$, $z = 0$. Then the velocity of the origin is $+aq$, hence the screw gives rise to a force at the origin equal to qY_q along the axis of y and couples $-aqN_w$ and $+q$ (ang. mom. $\div g$) about the axis of z. The sum of the moments of this system about a line parallel to the axis of z whose equations are $x = x_1$, $y = 0$ is

$$q\,(+x_1Y_q - aN_w + \text{ang. mom.} \div g) \quad . \quad . \quad . \quad (28)$$

and this may be made to vanish by suitably choosing x_1 and a.

If x_1 is the co-ordinate of the centre of gravity and $x = a$, $z = 0$ the equations of the axis about which the aeroplane is turning, and these are so chosen as to make the coefficient of q in the above expression vanish, *the machine will not tend to dip up or down in turning curves.*

By taking the turning axis to be horizontal it would similarly be possible to choose its position and that of the screw so that *lateral rotations are not set up when the system is being steered in a vertical plane.*

It will be seen that the resistance derivatives depend on the slip ratio of the air-screw, but if this is maintained constant everything is proportional to the speed, so that the balance is unaffected by changes in the speed. The main difficulty of such an attempted balancing would of course arise from the fact that the position of the turning point is liable to vary considerably, especially in aeroplanes without or with insufficient fins or auxiliary surfaces, and further, it would probably be difficult in any case to make the turning centres coincide for vertical and horizontal steering. I consider, however, that the formulæ indicate the possibility of reducing the disturbing effect of the air-screw by suitably choosing its position with reference to the centre of gravity and the range of positions commonly assumed by the turning centre in ordinary steering. It will at any rate be seen that the disturbing effects depend to a considerable extent on the position of the air-screw, and I desire to point out, in conclusion, the importance, especially in a formal study of problems of this kind, of examining results indicated by theoretical considerations entirely apart from any questions of their practical applicability. Any attempt to limit the investigation by the exclusion of such possibilities would necessarily render the presentation of the results incomplete and more difficult to understand.

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INHERENT CONTROLLABILITY OF AEROPLANES.

NOTES ARISING FROM PROFESSOR BRYAN'S WILBUR-WRIGHT MEMORIAL LECTURE.

BY LEONARD BAIRSTOW, A.R.C.SC., ASSOCIATE FELLOW.

The first direct statement of the problem of controlling an aeroplane, without the permanent exercise of physical effort by the pilot, appears in this paper. During the state of transition from straight flying to flying in a circle, a pilot exerts a force to pull over the rudder and in the usual case it is necessary to maintain some force in order to make a continuous turn. Professor Bryan proposes the design of an aeroplane such that turning having been started, maintenance of turning is obtained without any effort being necessary to keep the rudder in position. So far as the pilot is concerned the same result would be obtained by balancing the control surfaces, but the effect on the aeroplane is quite different. In the case of balanced controls a lateral force is still exerted by the rudder and so stresses are introduced in the tail structure of the aeroplane; on the other hand, in an inherently controllable aeroplane, there is no lateral force on the rudder when turning and, therefore, no stresses in the tail.

The practical importance of the main idea underlying the principles of "inherent controllability" is immediately obvious for the case of large and consequently heavy aeroplanes. Unless some such effect be obtained, a point will soon be reached at which the muscular strength of a pilot must be reinforced by a servo motor and it is possible that to deal with heavy gusts such reinforcement will come moderately early into the position of importance. On the other hand, inherent controllability, or the balancing of control surfaces, or both in combination, may postpone the necessity and so avoid a certain amount of complication of mechanism.

Object of the Present Notes.

It is proposed in the following notes to attempt something towards bridging a gap between the purely mathematical analysis of Professor Bryan and the practical instincts of the engineer. The latter have been expressed by Mr. Lanchester in a note to the AËRONAUTICAL JOURNAL and the reply of Professor Bryan, published at the same time, contains some startling conclusions. The conclusions were not different from those given in the lecture, but the emphasis laid on them in the reply to Mr. Lanchester gives them added importance.

The main difference of opinion arises from the statement of Professor Bryan "that turning without banking ought to be correct provided that a reasonably proportioned aeroplane can be constructed embodying the necessary conditions and at the same time possessing 'inherent controllability.'" To the practical man contemplating turns requiring angles of bank of 45° or more, the possibility of making equally rapid turns without banking would probably never occur, for there is nothing in a modern aeroplane which can produce conveniently the necessary opposition to centrifugal force. Nor is it in the least certain that anything could be introduced efficiently which would remedy the deficiency. What is certain, is that such rapid rates of turning, known to be of great importance in flight manœuvres, were not contemplated by Professor Bryan in his lecture. This

brings the writer of the present notes into line with Mr. Lanchester in the opinion that certain conclusions of the lecture are unpractical.

The Relation Between Assumed Conditions and Actual Practice.

In writing his paper Professor Bryan starts with certain clearly stated assumptions and deduces by strictly logical mathematical processes certain conclusions, and it may be that the mathematical processes have more attraction for him than the degree of accuracy with which the results can be directly applied to actual aeroplanes. In his book, "Stability in Aviation," a similar position was taken, but nobody with a knowledge of the course of aeroplane construction will question the importance of this book in the development of inherent stability in modern flying machines. Between the mathematical analysis in a comparatively rigid form and the construction of an inherently stable aeroplane there exists an appreciable amount of work by other people. The ideal plane of Professor Bryan does not exist in practice, and actual aeroplanes are provided with bodies and undercarriages not directly included in the initial assumptions of "Stability in Aviation." Moreover, the special devices for obtaining stability proposed by Professor Bryan are developed mainly in reference to a constant speed aeroplane and are in that respect appreciably simpler than those existing in modern machines with a large speed range. These restrictions apply with perhaps greater force to Mr. Lanchester's work on stability, which is generally far less complete than that given in "Stability in Aviation"; agreement, where final conclusions have been reached by both methods, is however satisfactory. Some of the original assumptions made by Professor Bryan, such as that of narrow planes to avoid unknown changes in the position of the centre of pressure, and others which need not be enumerated, were clearly the result of lack of experimental data. The academic character so given to the main analysis, tends, unfortunately, to hide the real value from the severely practical man. It appears to the writer that there is great need for a class of worker* between the two extremes of pure mathematician and the strictly practical man, a class which needs the courage of its convictions since it is always in danger of falling between two stools. To this class falls the duty of adapting the mathematical and academic material to practical needs and the presentation of results in such a form that they become more immediately available to the practical man. In the region of aeronautics, and particularly in relation to stability, this position has been filled to some extent by the activities of the staff in the Aeronautics Division of the National Physical Laboratory, and it is again in this capacity that the present notes are attempted.

Detailed Survey of the Lecture.

Having explained as far as possible the general attitude of the writer towards the study of mathematical papers on aeronautical subjects, it is now proposed to follow through Professor Bryan's paper in some detail, pointing out the assumptions made and where they involve differences from aeroplane practice. It will readily be realised after what has been said that the writer is personally satisfied as to the accuracy of Professor Bryan's mathematical steps; this necessarily involves the corollary that where any difference of opinion occurs it must lie in the applicability of the initial assumptions to practice.

Balancing Flaps.

Professor Bryan makes an important restriction throughout his Wilbur Wright Memorial Lecture, when he avoids the use of balancing flaps. No proposal to discard such flaps is seriously contemplated in practice since it is neces-

* [An aeronautical form of the "scientific middleman" for which Grant Allen contended.—EDITOR].

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sary to deal with gusts and in preparing to alight to make what may be described as "unnatural"* motions.

If balancing flaps are present for any reason whatever the conclusions of Professor Bryan's paper are appreciably modified by practical considerations. For instance, it is *practically* immaterial whether a couple be very small or really zero. *Mathematically* a distinction can be and is drawn, and it will be found that in the author's view such a distinction has led to the more startling propositions in Professor Bryan's paper; without such restriction, practical inherent controllability can be obtained without the use of complex wings and without the necessity for appreciable modification of the existing method of turning; somewhat similarly the advantage of turning without banking appears to lie mainly in the simplification of the mathematical analysis and it is not clear that it leads to any simplification of control as it appeals to the pilot.

Definition of Terms.

In order to simplify reference to aeroplane motions it is desirable to define such terms as "turning centre," "side-slipping," etc.

Suppose that an aeroplane is turning in still air in a horizontal circle of which the centre is O (Fig. 1). The "turning centre" TC is defined as the

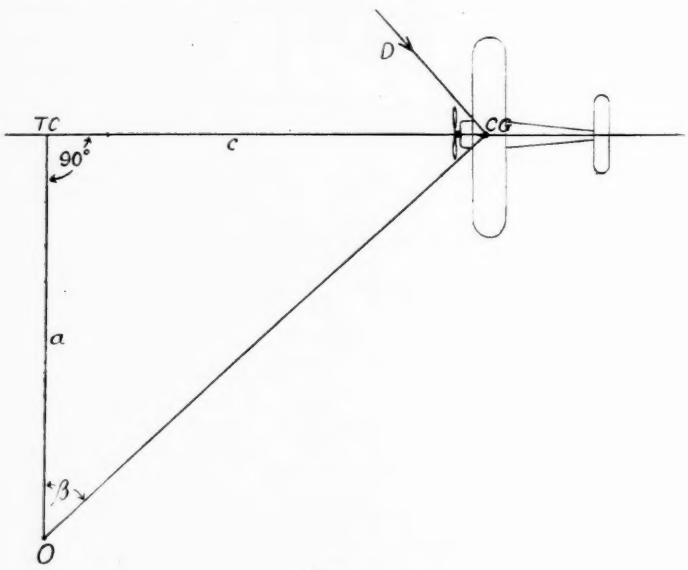

FIG. 1.

point on the axis of the aeroplane at which the perpendicular from O cuts that axis. If a line be drawn from O to CG, the centre of gravity, the aeroplane will be seen to be moving at right angles to such line and the relative wind will appear to come from D; the angle of yaw as usually defined is the angle β.†

* The word "unnatural" is here used in a sense corresponding with "natural bank" and is intended to imply that the aeroplane would not make such movements automatically, however perfectly it may meet its conditions of design.

† If the angle of bank, ϕ, is not zero, the angle of yaw is defined by the relation $\beta = \tan^{-1} \frac{c \cos \phi}{a}$

If TC coincides with CG the angle β will be zero and the aeroplane is then said to be not "side-slipping." A little consideration will show that this definition is somewhat arbitrary in the case of turning, as the pilot would feel the wind on one side of his face when sitting in front of CG and on the other if sitting behind, when the definition said "no side-slipping." If the "turning centre," *i.e.*, the point which is moving along the axis of the aeroplane, be used instead of CG, the pilot's sensations related to that point will be as above, and it will be clear that for any point such as TC Fig. 1, the side wind will appear to the pilot to come from the same quarter, no matter what the position of his seat within the limits permissible in an aeroplane. The direction from which the relative wind strikes the pilot is in itself unimportant, but since he is always near the wings, the properties of which depend on the angle of yaw, it is important to keep this angle within reasonable limits. Probably β should not exceed 10°. It then follows that within certain limits the TC may be varied, but that if far away from the CG either before or behind, lateral forces and disturbance of wing forces become important; these forces, etc., are considered in Professor Bryan's analysis on the assumption that the angle of yaw is small and any result not consistent with this is rejected. In other words, Professor Bryan has imposed restrictions on the possible values of the distance of the turning centre from the centre of gravity.

The General Equations—Lateral Group.

Turning to equations (5*w*), (5*p*), (5*q*), page 56, AERONAUTICAL JOURNAL, April-June, 1915, we have the following lateral group of equations:—

$$-\frac{W}{g}\cos\phi a\Omega^2 + W\sin\phi = -\Omega\cos\phi\,(cZ_w + Z_q) - Z_1 \quad . \quad . \quad . \quad (5w)$$

$$(B - C)\sin\phi\cos\phi\Omega^2/g = -\Omega\cos\phi\,(cL_w + L_q) - L_1 \quad . \quad . \quad . \quad (5p)$$

$$F\sin\phi\cos\phi\Omega^2/g = -\Omega\cos\phi\,(cM_w + M_q) - M_1 \quad . \quad . \quad . \quad . \quad (5q)$$

and these are to be satisfied if the aeroplane is to be in equilibrium when turning with angular velocity Ω. a is the distance of the turning centre from the centre of the circle in which the aeroplane is turning and c is the distance of the turning centre in front of the centre of gravity. W is the weight of the aeroplane and g the usual acceleration due to gravity.

The Longitudinal Equations not considered.

The longitudinal equations of motion will not be considered in these notes, although as Professor Bryan points out, they are affected by the circling motion; a consideration of these would lead to inherent longitudinal controllability and the complete problem would become more complex. For the not very rapid rates of turning contemplated in the original paper the present separate treatment of lateral motion appears to be sufficient; on the other hand, it is largely on account of the simplification introduced into the longitudinal conditions of equilibrium that Prof. Bryan favours a solution of the problem in which the angle of bank ϕ is zero.

Consider equations (5*w*), (5*p*) and (5*q*) in detail. The first term $-\frac{W}{g}\cos\phi a\Omega^2$, is the component of centrifugal force along the axis of z, *i.e.*, along the lateral axis. Whilst the centrifugal force is horizontal, the axis of z is inclined at the angle of bank ϕ. The second term $W\sin\phi$ is the component of the weight of the aeroplane along the same axis. On the right hand side of the equation the term having cZ_w as a factor is the lateral force due to side-slipping of the aeroplane without rudder and can clearly be varied by putting in fins or altering the side surface of the body of an aeroplane. The term containing Z_q is the lateral force on the aeroplane without rudder due to turning and is usually very small. The term Z_1 is the lateral force on the rudder alone and it is one of

the particular objects of the inquiry to make Z_1 zero by a suitable choice of Z_w and Z_q.

The right hand side of (5p) is very similar to that of (5w) the terms being the rolling moments due to side-slipping and turning and L_1, a rolling moment due to controls. The balancing flaps are the practical means of producing L_1, and as Prof. Bryan proposes to do without them it is necessary for him to satisfy the condition $L_1 = 0$. The left hand side of (5p) gives a term known as the "centrifugal couple" and is the source of much trouble in the succeeding analysis. The moment of inertia about the vertical axis (B) is, in actual aeroplanes, almost necessarily greater than that about the lateral axis (c); due to the turning of the aeroplane this difference in moments of inertia produces a couple tending to retard the lifting of the outer wing if the banking is correct for the turn. This couple is of little practical importance as compared with any which can be produced by balancing flaps.

Equation (5q) is interpreted very similarly. On the right hand side we have the yawing moments due to side-slipping and turning on the machine without rudder and both of these may be important practically. The yawing moment M_1 is mainly due to the rudder, but the use of balancing flaps in the actual aeroplane may introduce appreciable effects if the movement is large. As a moment due to the rudder almost inevitably involves a lateral force near the tail of the aeroplane, the conditions of inherent controllability can be seen to require that $M_1 = 0$. On the left hand side of the equation occurs an inertia couple depending on the product of inertia F. In practice this couple is small and unimportant.

The mathematical statement of the physical problem is now that of finding of form of aeroplane in which Z_w, Z_q, L_w, L_q, M_w and M_q shall have such values that Z_1, L_1 and M_1 shall be zero for small values of Ω; in proceeding to the special cases considered, use will be made of numerical examples.

The aeroplane will, for this purpose, be assumed to have a weight of 2,000 lbs. and to be flying at a speed of 100 ft./sec. and at an angle of incidence of 0.1 rads. (5·73°). The approximate total wing area will be 400 sq. feet with a span of 40 feet. As an indication of the value of Ω (the rate of turning) it may be mentioned that for one turn in 20 secs. $\Omega = 0.3$ approx. and the corresponding natural angle of bank is 45°.

Straight Planes and Vertical Rudders.*

This is the simplest of all possible types of aeroplane. It consists of flat main planes and flat elevator with a single flat fin for a rudder. There is neither body nor under-carriage. Without rudder, such a combination has clearly no resistance whatever to side-slipping. Due to turning there is a rolling couple arising from the higher velocity of the outer wing and this tends to bank the aeroplane. Accompanying the rolling couple is a yawing couple produced by the difference of drag on the two wings. These are the only couples which occur and the formulæ for numerical amount are given by Professor Bryan.

Putting these values into the equations of motion it is immediately clear from (5w) that the angle of bank corresponding with equilibrium is that natural to the turn. On attempting to satisfy the equation (5p) and (5q) it is found that even in the most favourable case the rolling couple due to the more rapidly moving outer wing is 100 times as great as the centrifugal couple and cannot therefore be balanced for any reasonable difference in the moments of inertia. Similarly, the yawing moment is too great to be balanced by any reasonable value of the product of inertia.

* Aeronautical Journal, p 56. Chapter II.

Even if the rudder is used Professor Bryan shows that the equation for rolling moments is still intractable.

Straight Planes with Boxed-in Ends or Vertical Partitions.*

The effect of the introduction of vertical surfaces is to give a value to Z_w, the lateral force due to side-slipping. It is no longer necessary to fly at the angle of natural bank in order to satisfy equation (5*w*). On the other hand, the difficulties relating to rolling and yawing due to difference of velocity on the two wings when turning are not substantially affected.

Raised Fins or Rudders.†

The suggestion is made that fins above the *CG* or bent up wings (dihedral angle) should be introduced to deal with the rolling moment equation. Mathematically this introduces the expressions L_w and M_w into the conditions of equilibrium, L_w being the rolling moment due to side-slipping and M_w the yawing moment due to side-slipping.

Bent-up Planes.

The case of bent-up planes is considered in Chapter III., and although the new term of importance is L_w, this expression does not appear explicitly in the analysis. For good mathematical reasons the axes of co-ordinates have been changed and L_w loses its identity as part of L_q, which now includes both the effects of turning and of side-slipping. Section VI. is very general and will not be discussed here, but attention directed immediately to VII. The aeroplane now consists of wings having a simple dihedral angle, each section of which is flat, with an elevator and rudder behind the wings. There is still no body.

Working out a numerical case, it is found that for rapid turning ($\Omega = 0.3$) the angle of bank differs little from that "natural" to the turn and that the values of the rolling moment and yawing moment can be reduced to very small amounts by a suitable choice of the dihedral angle without excessive side-slipping being needed for equilibrium. For instance, with a dihedral angle 3° the turning centre can be brought to 50 feet from the *CG* (behind) or half this for 6°. The corresponding angles of yaw would be roughly 10° and 5° respectively. We here come across the one great divergence in the paper between mathematical rigidity and the practical needs of the problem. If the centrifugal couple be neglected in the rolling moment equation the distance of the turning centre behind the centre of gravity is 53 feet, whilst if it is included the value is 54 feet. Similarly, the couple arising from the product of inertia F has little effect on the position of the turning centre and moreover F can be reduced to zero, in which case it has none.

If then we neglect a couple which is about 2% of that arising from the difference in velocity of the two wings, the conditions of inherent controllability are practically satisfied by an ordinary dihedral angle and normal tail planes. There is no necessity for the introduction of the idea of no banking, other than that of mutual effect between longitudinal and lateral balance, an effect not now under discussion.

An exceedingly small movement of the wing flaps would provide the required couple to make this method strictly applicable, and the whole of the rest of Professor Bryan's analysis arises from a consideration of this practically negligible rolling moment. It will be seen that for a dihedral angle of 6° or thereabout the side-slipping would be equivalent to the relative wind coming in from the inside of the turn at an angle of about 5°.

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* AERONAUTICAL JOURNAL, p. 58. Section IV. † *Ibid.* p. 60. Section V.

The differences between the ideal aeroplane of this section and an actual aeroplane are not of such a character as to lead to the method of producing inherent stability being unpractical.

It will be seen that the balance of rolling moments is obtained by overbanking and consequent side-slipping inwards, so introducing a righting moment due to the dihedral angle of an amount sufficient to counteract the upsetting moment due to the more rapidly moving outer wing. At the same time the yawing moments due to side-slipping and turning are counterbalanced to a large extent and the balance could be modified to some extent by changes in the side surface of the body at points not far from the main planes.

Complex Wing Forms.

From a practical point of view then, the further analysis given in Prof. Bryan's paper would be considered unnecessary. It deals with complex wing forms, having varying angles of incidence, with the inclinations of the wings to the lateral axis varying, with the chord variable and the wings swept backward or forward. All this complication is introduced as a result of an endeavour to obtain mathematical rigidity and leads to some very interesting general theorems. It is not proposed to examine these further proposals in great detail, but to make one or two general remarks. If turning without banking is attempted, side-slipping outwards will occur and the type of balancing proposed above would fail completely; if an attempt were made to readjust the conditions by introducing a negative dihedral angle, trouble would be encountered in relation to stability. To produce the necessary force to balance the centrifugal tendency, something in the nature of vertical planes or curtains would be required, with, of course, some loss of efficiency even in straight flights. Instead of considering only simple cases the mathematics has been kept so general that every little restriction on the form of wing occurs, and it is this generality which will probably allow an aeroplane to be made which is both " inherently controllable " and " inherently stable."

Professor Bryan, in the reply to Mr. Lanchester, gives a proof that stability is not incompatible with inherent controllability and this is accepted as correct not only for the special aeroplane of the mathematics but also for the real aeroplane.

The Effect of Varying Angle of Incidence and Speed Range.

The balance on which inherent controllability depends is however modified by change in the angle of incidence of the aeroplane, since the rolling and yawing moments due to side-slipping and turning are greatly modified by such change. It then remains for further inquiry how far " inherent controllability " is possible over the whole speed range of a modern aeroplane and whether the amount of side-slipping involved is or is not prohibitive. There is also a definite practical point as to how far forces on the fin and rudder are objectionable features in view of the fact that the aeroplane must be made strong enough to stand against gusts which strike it whilst the rudder is held fast.

The problem of inherent controllability seems to possess its main interest in connection with the muscular fatigue of the pilot, and so many additional considerations then enter into the problem that further discussion must be deferred to a later period.

Turning Without Banking is not a Practical Proposition.

The conclusion is then reached that the condition of no banking is not an essential part of Professor Bryan's analysis and that " inherent controllability "

can, for practical purposes, be obtained without resort to complex wing forms. It is also concluded that no practically acceptable solution of the problem of aeroplane motion can be based on the possibility of turning without banking.

These conclusions, although superficially different from those of Professor Bryan, do not in the writer's opinion involve as a consequence any single point of disagreement with Professor Bryan as to whether the conclusions in the lecture do correspond with the assumed forms of aeroplane and limited rates of turning.

[NOTE.—Mr. Bairstow having sent an advance copy of the foregoing paper to Professor Bryan, received in reply a letter from which extracts are appended, and which will doubtless make much clearer the point of view from which Prof. Bryan has approached the subject.—EDITOR.]

BANGOR,
21st Feb., 1916.

MY DEAR BAIRSTOW,

I am in entire agreement with your notes on "Steady Motion in a Circle." The explanatory abstract which you have written is exactly the kind of contribution which is most suitable for the AËRONAUTICAL JOURNAL. In regard to papers of a "cut and dried" mathematical character, such as my recent contributions to the Aëronautical Society and the even more formidable ones which I am shortly about to submit to them, I think you know quite well that ever since my article in "Science Progress," in 1897, I have lost no opportunity of urging mathematicians of the orthodox type to take up these problems, and had they done so, such papers would have first seen the light in a more purely mathematical atmosphere than the Aëronautical Society. This plan would have had the advantage that the subject could have been thoroughly and systematically developed independently of practical considerations, and both experimenters and engineers would have been free to pick out such sections as they found useful and to discard the rest.

The reason I adopt these rigorous mathematical methods is not because I have any liking for them, but because nobody else has done this work and I want to learn what it leads to. I should far prefer to learn something about practical aeronautics, but any attempt to do so would only add to the collection of unsolved problems presented by aeroplane motions.

The purely mathematical literature of the aeroplane developed on these lines ought to be at least as extensive as our existing literature on hydrodynamics, and it will stand in about the same relation to practical aeronautics as that subject stands to the practical problems of hydraulic engineering and naval architecture. The systems of narrow planes gliding at small angles are the simplest representations of aeroplanes and stand in somewhat the same relation to the latter that the infinitely extended mass of incompressible perfect fluid bears to actual liquids.

I enclose a cutting from "Nature,"* which contains a sample of the kind of thing I have been preaching to orthodox mathematicians for years, but with no result, and consequently I consider that there is no alternative but to ask the Aëronautical Society to publish my papers, despite their somewhat academic treatment.

Had not Messrs. Macmillan undertaken to publish my "Stability in Aviation," under the editorship of Prof. R. A. Gregory, I do not think it would have been

* NATURE. January 22nd, 1916, pp. 575-6—not here reproduced.—EDITOR.

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possible *at that time* to find any suitable medium of publication for them, and in that case the only developments of this method would have been those published by Prof. Reissner in Germany. . . .

A serious disadvantage of the existing conditions is the risk of giving rise to premature discussions on immature theories. . . . Mr. Lanchester's criticisms have caused the conditions for turning without banking to assume an exaggerated amount of importance. . . .

My expression "provided that a reasonably proportioned aeroplane can be constructed" was probably a sufficient limitation of the extent of my proposals, and I take it that what you now say is that the condition in question is not practicable. I am not at all surprised at this, for you will see in my paper that even the equations are rather difficult to satisfy, and when we come to investigate the problem of disturbed motions I expect the system in question will be knocked out. . . .

The real point of difference between myself and Mr. Lanchester is that I consider that the theoretical non-banking system plays an important part in the *formal* study of the problem of circular motion, since it represents an extreme case, the opposite extreme being represented by the laterally unstable systems contained in the earlier part of my paper. Referring to your figure where $OT_c = a$ and $T_c C_g = c$, you will see that in the non-banking system c is constant, whereas in what I call the straight-plane systems a is constant. The result is to give excessive side-slipping in the former case in turning curves of small radius and in the latter in turning curves of large radius. The fact that my investigation does not introduce the effects of ailerons must not be taken as implying any expression of opinion regarding their use. By investigating what would happen in a system without them, it is possible to see more clearly *why* and under what conditions their use comes in.

I fully appreciate the immense amount of work you and the late Mr. Busk and other members of the N.P.L. and the Government Committee must have done. Even to read my book and try to make anything at all out of it must have been a pretty troublesome job. . . .

I am in correspondence with two colleagues and hope we may soon tackle the formal theory of initial effects of disturbances; when this is done I may possibly be ready to look at aeroplanes themselves. . . . I am afraid we shall have to start with considering small disturbances only, and that the "practical men" may again have their knife into us in consequence. If so, the only result will be to cause delay in attacking the problem of finite disturbances which seems not altogether out of reach, and which I believe to be very important.

I am very glad to have your authoritative opinion about the combination of inherent stability and controllability, because I have been told over and over again that inherent "stability is of no practical value whatever, as an aeroplane is subject to such continual changes of velocity."

I think you and your colleagues have accomplished an extraordinary feat in getting inherent stability into a sufficiently practical form to be incorporated in military aeroplanes. I am afraid it is important for us all to keep going at aeroplane problems whether immediately practicable or not, because in the pre-war days Reissner and Bader were running us pretty hard on the mathematical side. It is a great mercy that we *have* got a National Physical Laboratory with people like you at work in it, but it must not be forgotten that the idea of establishing such an institution was suggested originally by the Berlin laboratory,

which has far more money at its disposal, and the Germans are probably putting their best brains into improving their aeroplanes.

Unless you have any wish to the contrary, I should be glad if you can send this letter on to the Aëronautical Society for publication with your notes, and it might be interesting also to send them the cutting from "Nature." . . .

G. H. BRYAN.

THE FAN DYNAMOMETER.

SOME NOTES ON TESTS.

BY J. LAWRENCE HODGSON, ASSOCIATE FELLOW.

The investigation described below was undertaken in order to obtain a more accurate knowledge than can be obtained from any published results as to how the resisting torque due to the blades of a fan dynamometer varies with their shape and size, with the diameter of the fan, and with the density of the fluid in which the blades rotate.

Description of the Apparatus.

The blades used were $\frac{1}{2}''$, $1''$ and $2''$ square, and $\frac{1}{2}''$, $1''$ and $2''$ in diameter. They were cut from smooth German silver sheet .012″ thick, and the edges were left sharp and at 90° to the face of the blade.

All the tests were carried out with the fan rotating in water; with the exception of one test, which was taken with the fan rotating in air.

In the water tests, the vertical spindle carrying the fan was rotated in a covered tank of still water 20′ square and 5′ deep.

This spindle was driven by a small air turbine attached directly to it, and the driving torque was measured by means of a manometer which indicated the pressure at the turbine jet. The relation between the driving torque and the readings of the manometer was determined by a series of brake tests taken at various jet pressures and turbine speeds.

The plane of the fan arms was 10″ below the surface of the water, the coefficient of resistance having been found to be constant for the largest diameter of fan used for all depths of immersion greater than 6″.

The apparatus and the observer were carried on a platform which bridged the tank.

In every case, before a reading of the speed of rotation was taken, the fan was rotated under constant torque until there was no further increase in the speed.

The mean temperature of the water during the experiments was 50° F.

Analysis of Results.

A separate series of tests established the fact that the torque due to the arms (μ_a) was given with sufficient accuracy by the formula:—

$$\mu_a = \cdot 35 \times 10^{-8} \sigma N^2 r_a^4 t W * \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In order to increase the accuracy of the measurement of the torque due to the blades, μ_a was made as small as possible by using arms of stream line form.

The tests were analysed on the basis that the resisting torque (μ_b) due to the blades alone was given by the equation:—

$$\mu_b = K N^2 r^3 S^2 W * \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where K is a coefficient which varies with the ratio r/s and with the shape of the blades, but which is constant for similar fans.

The outside radius r is used in this equation in preference to any other linear dimension which varies with the diameter of the fan, such as the radius of the

* A list of the symbols used is given at the end of the Paper.

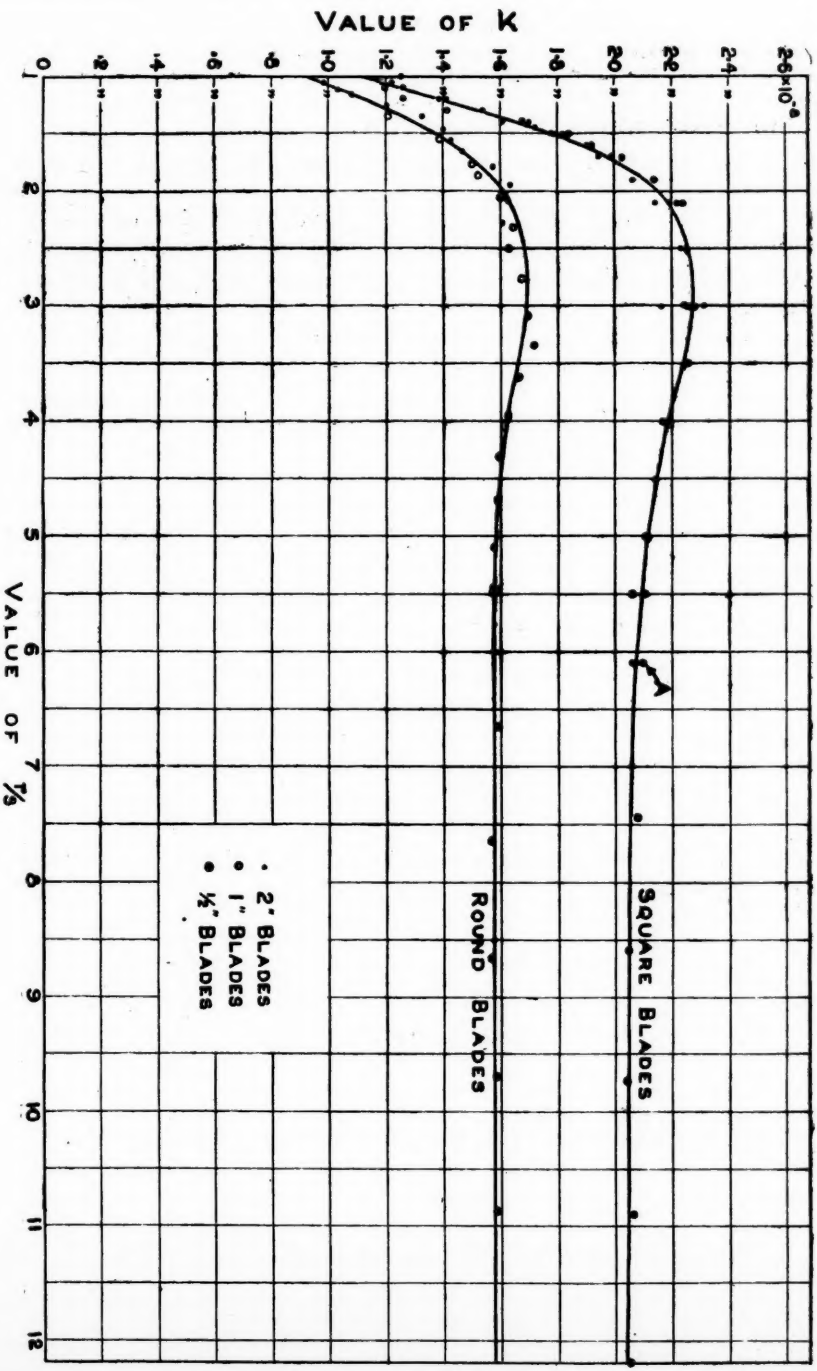
VALUE OF K
·0
·2
·4
·6
·8
1·0
1·2
1·4
1·6
1·8
2·0
2·2
2·4
2·6×10⁻⁶
1
2
3
4
5
6
7
8
9
10
11
12
VALUE OF 7/8
· 2" BLADES
• 1" BLADES
• ½" BLADES
ROUND BLADES
SQUARE BLADES

Fig. 1.

Centre of Gravity or of the Centre of Pressure of the blades, because it is the linear dimension which is most easily measured.

The Ascertained Value of "K."

Fig. 1 shows the values of K plotted against the ratio r/s, as determined from all the experimental results.

It will be noticed that for any given value of r/s the value of K does not vary appreciably with N^2 or with the size of the blade.* Also that when r/s is large, K approaches a constant value for each shape of blade; and that the limiting value of K for the circular blades is about .7854 times less than that for the square blades.

Comparison with N.P.L. Results in Air.

It will be obvious that this limiting value of K should correspond to the coefficient of resistance for plates exposed normally to the wind.

The point marked N.P.L. in Fig. 2 shows the value of K in the limit when r/s equals infinity, as calculated from the value .0032 given for plates 4′ 0″ square and upwards in the 15th report of the Advisory Committee.

It will be seen that the value there given for large plates in air is only 2.4% greater than the value now obtained for blades $\frac{1}{2}$″ square in water. The value deduced from the author's tests must in any case be less than that obtained from plates exposed normally to the wind or moving in a straight line through undisturbed fluid, owing to the rotation of the fluid set up by the fan.

The Same Blade Rotated in Air and Water.

The point marked A in Fig. 1 is the value of K obtained from 1″ blades in air. It agrees almost exactly with the value of K obtained by rotating the same blades in water.

The point marked P in Fig. 2 corresponds to the value of K as deduced from Mr. Poppe's formula, which is given in a paper by Messrs. Morgan and Wood, published in the "Transactions of the Inst. of Automobile Engineers," p. 530-550, Vol. VII. Mr. Poppe's tests were made on fans for which the value of r/s varied from about 2.5 to 10, so that the value of K deduced from his formula agrees very fairly with the dotted curve.

The other points plotted in Fig. 2 are calculated from Messrs. Morgan and Wood's tests, as given in the paper above referred to. These tests were taken on blades 6″ and $8\frac{1}{2}$″ square, and would seem to indicate that the author's results, when applied to large blades moving in air, are about 3% too low.

It is probable that the dotted curves in Fig. 2 represent values of K which are not more in error than 1% or 2% at any point when applied to blades of the dimensions used in engine testing.

The Application of the Results to Fan Calculations.

The H.P. absorbed by any fan can be readily calculated by substituting the value of K as given by these dotted curves in the equations tabulated at the end of the paper.

If high accuracy is desired the fan should be mounted so that there is no obstruction to the free circulation of air in its vicinity.

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* In the author's experiments N varied for any one fan over a range of 1 : 3 ; in Messrs. Morgan and Wood's experiments over a range of 1 : 2. These ranges are too small to prove that the N^2 law is followed exactly.

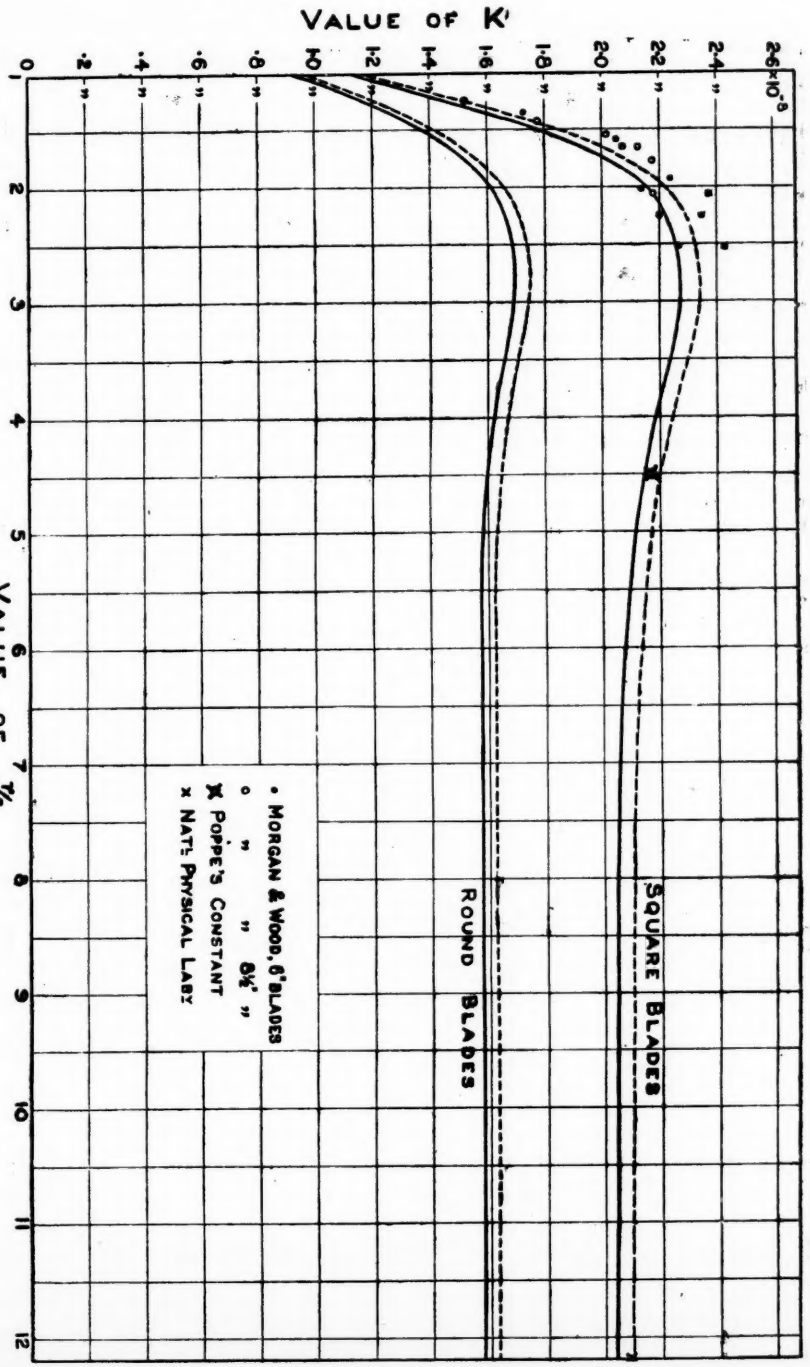
VALUE OF K'
0
·2 "
·4 "
·6 "
·8 "
1·0 "
1·2 "
1·4 "
1·6 "
1·8 "
2·0 "
2·2 "
2·4 "
2·6 ×10⁻³
2
3
4
5
6
7
8
9
10
11
12
VALUE OF V/n
• MORGAN & WOOD, 6' BLADES
o " " 8½' "
X POPPE'S CONSTANT
× NAT'L PHYSICAL LAB'Y
ROUND BLADES
SQUARE BLADES

FIG. 2.

The error caused by such obstructions is very fully dealt with in Messrs. Morgan and Wood's paper.

The Ratio of Blade Area to Radius of Rotation.

In fan brake dynamometers, as usually constructed, it is customary to make the value of r/s small.

It will be seen, however, on reference to Fig. 1 that the experimental accuracy is least for the small values of r/s.

Further, the shape of the K—r/s curve is more easily defined from theoretical considerations when the ratio of r/s is large.

The curve can therefore be drawn with much greater certainty at these values.

For these reasons it is preferable to design the fans so that the ratio r/s is not less than 2.5 or 3.

The Shape of Blade.

Then again, although square or rectangular blades are universally used, round blades are to be preferred, as they are easier to make accurately and require no special care in alignment when being fitted to the arms.

It is worthy of note that when *comparing* the performances of a number of petrol engines by means of the same dynamometer, no correction need be made for variations in the temperature of the air or in the barometric pressure, as these factors affect resistance of the fan and the volume of air passing through the engine at any given speed in almost exactly the same ratio.

APPENDIX.

A summary of the equations and symbols used is given below.

FIG. 3.

$$\mu_a = .35 \times 10^{-8} \sigma N^2 r_a^4 t W \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\mu_b = K N^2 r^3 S^2 W \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\mu = \mu_a + \mu_b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\text{H.P.} = \frac{\mu N}{63{,}000} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Where

W = Density of the fluid in which the fan rotates in lbs. per cubic foot.
= 62.33 for water at 50° F.
= $2.7 \frac{p}{T}$ for air.

p = Abs. press. in lbs. per square inch.
= Barometer reading in inches of mercury × .489.

T = Abs. temp. in degrees F.
= (459.5 + t° F.)

μ_a = Torque due to arms alone in inch lbs.

μ_b = Torque due to blades alone in inch lbs.

μ = Total torque in inch lbs.

N = Revs. per min.

S = Length of side of square blade, or diameter of circular blade, in inches

r = External radius of the fan in inches.

r_a = Radius of arms in inches.

t = Thickness of arms in inches.

K = Coefficient depending upon the value of r/s and the shape of the blade.

σ = Coefficient depending on the section of the arms, as given below.

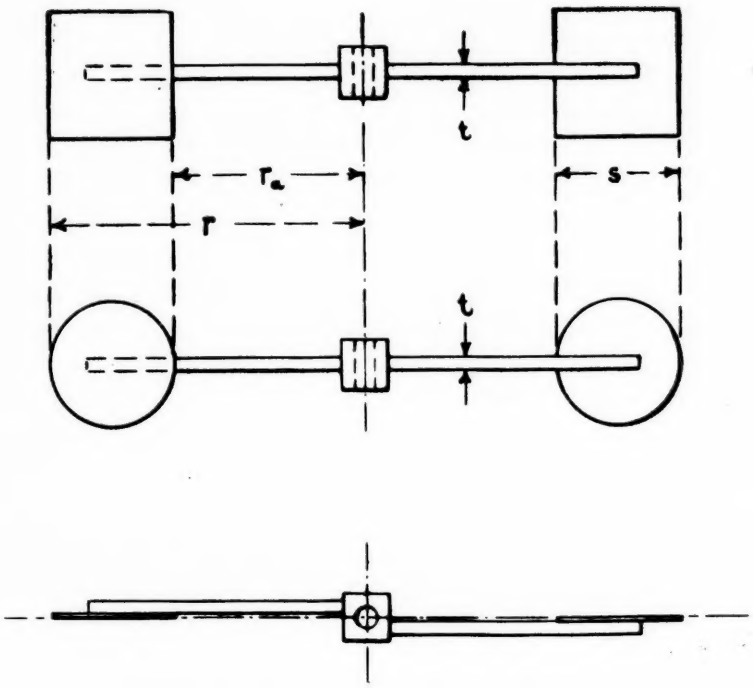

FIGURE 3.

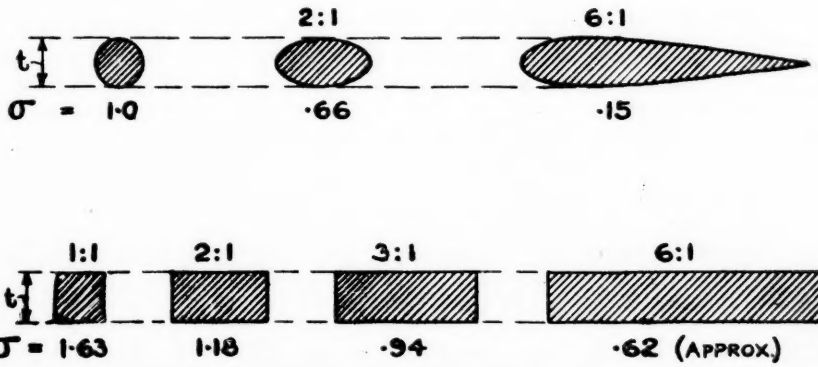

FIGURE 4.

It will be obvious that the equation

$$\mu_b = KN^2r^3SbW \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where K is the coefficient for square plates given by the dotted curve in Fig. 2, will be at any rate approximately correct for rectangular blades of radial width S and breadth b inches.

An analysis of the tests made by Messrs. Morgan and Wood on rectangular plates, for which S varied from .7 times to twice b, shows that this equation very fairly represents the whole of their experimental results, and may be taken as practically correct.

In conclusion, the author would like to thank Mr. F. Gray, who carried out the main experimental work upon which these notes are based, and also Messrs. Geo. Kent, Ltd., of Luton, at whose works the tests were made.

NEW BOOKS AND PUBLICATIONS.

Catalogus der Bibliotheek van der Koninklijke Nederlandsche Vereeniging voor Luchtvaart. Koninklijke Nederlandsche Vereeniging voor Luchtvaart, Amsterdam. 1915. pp. 55.

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Science and Technology: With reference to the War in its Technological Aspects. London. 1916. "The Athenæum." pp. 79. Price 2s. 6d.

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Notices of the Aëronautical Society of Great Britain.

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Those Members who have not yet paid their subscriptions for the current year are reminded that these fall due on the 1st January each year.

Wilbur Wright Lecture, 1916.

The Fourth Wilbur Wright Memorial Lecture will be delivered on Tuesday, June 6th next, at three p.m., in the Theatre of the Royal Society of Arts, John Street, Adelphi, W.C.

The title is "The Life and Work of Wilbur Wright," and the lecturer, Mr. Griffith Brewer, A.F.Aë.S.

The Right Hon. the Lord Montagu of Beaulieu will preside.

Tickets will shortly be available for distribution from the Offices of the Society, 11, Adam Street, Strand, W.C., to which address applications should be made.

Edward Busk Studentship in Aeronautics.

Applications will shortly be invited for the above Aeronautical Research Studentship. The approved scheme provides that the award and control of the Studentship are to be in the hands of a Committee of whom three members are appointed biennially by the Aëronautical Society, and three by King's College, Cambridge (Edward Busk's old College), with two of the Trustees of the Fund.

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